SHAPE-FROM-SHADING AND PHOTOMETRIC STEREOSCOPY METHODS: A REVIEW

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ABSTRACT
Most of the existing works view the shape-from-shading problem as equivalent to a non-linear first-order partial differential equation in surface elevation. Recently, there is a new approach to this problem, namely the shape-from-shading using the heat equation. This paper reviews several works either using the first or the second approach. The previous approach is also implemented in solving the photometric stereo which is an extension of shape-from-shading problem. This paper also reviews the 2-D Leap-Frog algorithm for solving photometric stereo problem.

Keywords: Shape from shading, Photometric stereo, 2-D Leap-Frog algorithm, Lambertian model.

1 INTRODUCTION
Shape-from-shading is a problem of determining the shape of a smooth surface given a single image of that surface. The reconstructed shape can be expressed in several ways: depth, surface normal, surface gradient, and surface slant and tilt. The general solution of the shape-from-shading problem revolves around the image irradiance equation which is a non-linear first-order partial differential equation. Most of the methods for solving such a system specify the shape not by the height above a reference plane, but by surface orientation. There is a constraint called an integrability constraint such that neighboring orientations have to correspond to some underlying surface.

Some iterative methods which make repeated adjustments to surface orientation to improve the match between brightness predicted from the estimated surface and brightness actually observed maintains integrability of the estimated surface. However, since this turns out to be hard, other methods ensure only that neighboring surface orientations remain similar. These simpler methods have a common problem when one wants to estimate the shape in terms of a depth map, because usually no shape corresponds exactly to the computed field of normal.

This paper reviews several works to solve this problem. In addition, a novel approach to solve the shape-from-shading problem is also examined. This paper is divided into two main parts. The first part is about shape-from-shading, while the second one is about photometric stereo, which is an extension of shape-from-shading.

The outline of this paper is as follows. In Section 2, the paper briefly reviews the existing methods of shape-from-shading, each with the advantages and disadvantages. Section 3 presents alternative methods of photometric stereo and the analysis of the methods. Section 4 offers some conclusions and future research.

2 SHAPE-FROM-SHADING METHODS
There are several approaches in recovering shape given a single image of a surface. One approach is to estimate the surface orientation field, while the other approach is to estimate the height of a particular point in the reference plane. This section reviews three ways to reconstruct the shape: surface orientation estimate using a least-square technique, surface normal recovery using heat equation, and several methods for surface height recovery.

2.1 Surface Orientation Estimate Using A Least-Square Technique

According to Horn and Brooks [1], the shape-from-shading problem can be reformulated as one of finding a surface orientation field that minimizes the integral of the brightness error. However, minimizing the integral of the brightness error has an infinite number of solutions in terms of surface orientation fields. A regularization technique is then used and an extra term is added to the integral to
get a smooth approximation to a solution. Though, regularization technique does not always guarantee that the recovered surface orientation satisfies the integrability constraint. Moreover, the use of a regularization term tends to flatten and distort the solution.

To maintain the integrability constraint, a different parameterization for surface orientation was used by Horn and Brooks [1]. They introduced a penalty term which can be expressed in terms of the unit surface normal and its derivatives. Their initial tests indicate that they perform well.

However, this technique is sensitive to the initial surface normal directions. Also, the constraint that complies with the image irradiance equation and the constraint that complies with the local variation in the surface normal directions must be carefully balanced. Furthermore, fine surface detail is lost because the solution found is dominated by the smoothness model.

### 2.2 Surface Normal Recovery Using Heat Equation

According to Robles-Kelly and Hancock [2], the shape-from-shading problem can be posed as the solution of the linear heat equation subject to the constraint that the recovered surface normal satisfy Lambert’s law. A finite element method was then used to compute the components of the surface normal.

Lambert’s law consists of a constraint on the zenith angle between the light source direction and the surface normal direction. Therefore, the azimuth angle of the surface normal remains undetermined. One way to recover the azimuth angle is by using a smoothing method based on a heat flow analogy.

To do the recovery of the azimuth angle, Robles-Kelly and Hancock [2] introduced a scalar field whose gradient is the azimuthal component of the surface normal. In practice, it is only the polar angle of the gradient vector which determines the azimuthal angle of the surface normal. The heat equation provides a means of smoothing the azimuthal directions of the surface normal.

### 2.3 Surface Height Recovery

Besides solving the shape-from-shading problem by estimating the surface orientation or the surface normal, one can solve the problem by directly estimating the surface slopes. However, there is an integrability constraint such that the surface slopes correspond to a surface with second partial derivatives independent of the order of differentiation. One way to enforce this integrability constraint is by projecting the surface normal into the Fourier domain [3]. The surface height is recovered using an inverse Fourier transform. Clearly, this approach requires a finite basis function representation.

Another approach to solve the problem of surface height recovery is by reconstructing the surface height function from the pattern of local height differences. To be more specific, one may use the surface normal fields to estimate the height difference between each pair of pixel sites in the image. Once the height estimates have been calculated, then the surface may be recovered by embedding the surface normal into a one-dimensional Euclidean space perpendicular to the grid footprint [2]. This approach makes the link between surface normal and the surface that generated them becomes explicit.

### 3 Photometric Stereo Methods

Contrary to a single image shape from shading, photometric stereo is a problem of determining the shape of a surface given a triplet or pair of images. The images are obtained by consecutive illuminations of a given surface from three or two different point light-source directions. The three or two source photometric stereo for a Lambertian surface is modeled by a system of first-order nonlinear partial differential equations (PDEs).

Similar to the shape-from-shading process, there are two parts in photometric stereo: the gradient computation and the gradient integration. Noakes and Kozera [4] stated that the gradient can be uniquely expressed in terms of three or two images and light source directions. For the shape reconstruction process, the integrability condition has to be verified.

### 3.1 Linear Leap-Frog

One of the methods for verifying the integrability condition was proposed by Frankot and Chellappa [3] where the projection to the closest function expanded by Fourier series is used.
However, this method has an implicit requirement which is that the function has to be periodic.

Another method presented by Noakes and Kozera [4] does not need any a priori boundary condition or periodicity constraints. The method proposed, a 2-D Leap-Frog Algorithm, is based on the least-square optimization method applied to a non-integrable vector field prior to gradient integration. This iterative algorithm gives a sequence of sub-optimal solutions converging to the optimal one.

The following is an explanation of how the integrability condition be verified. The integrability condition is first transformed into its discrete analogue which yields a large system of linear equations in many unknowns. The least-square optimization method applied locally or globally solves perfectly the integrability problem. However, once the issue of computational complexity is raised, the advantage of resorting to the local approach, which is done by the 2-D Leap-Frog Algorithm, becomes apparent. Nevertheless, this method cannot remove the entire noise from the non-integrable vector field.

3.2 Non-Linear Leap-Frog

A more realistic assumption is that the photographic images are contaminated by Gaussian noise. This leads to a non-quadratic problem with many independent variables which depend on the image resolution. The most natural way to solve this problem is by global Gradient Descent. But regulating step-sizes can be tricky.

The non-linear 2-D Leap-Frog algorithm breaks the local optimization into a sequence of smaller optimization problems in fewer independent variables. Noakes and Kozera [5] have shown that the algorithm is convergent to the unique global minimum. However, a good initial guess is still needed. One way of making a good initial guess is by applying the linear Leap-Frog algorithm.

3.3 Parallel Leap-Frog

The 2-D Leap-Frog algorithm is one of the computationally feasible methods of performing non-linear optimization. Nevertheless, this process still takes a large amount of time using a single processor. A parallel implementation of the 2-D Leap-Frog Algorithm presented by Kozera, Cameron, and Datta [6] has shown that a high speed up and high efficiency can be achieved using a parallel method in a distributed shared memory environment.

The parallel Leap-Frog method uses a root processor to perform the computation along with the rest of the processors. Each processor communicates only with the processors connected to it. The topology used in this method is the grid topology of width one, which means processors have at most two processors to communicate with. However, the convergence will not be guaranteed if the individual processors are allowed to work entirely independently of each other. This is because the 2-D Leap-Frog algorithm converges to a globally minimal solution due to the many overlapping local minimizations. Furthermore, there must be synchronization at the end of the processing of the overlapping sub-squares buffers and at the end of each iteration processors.

4 CONCLUSION

This paper has reviewed several approaches for surface reconstruction using a single image (Shape-From-Shading Method) and more than one image (Photometric Stereo Method). Shape-from-shading is ill-posed if it is without any constraints such as integrability constraint, boundary constraint, smooth constraint, and so on. Photometric stereo is a well-posed problem since this does not need any additional constraints.

A novel approach to shape-from-shading was reviewed. This approach use the heat equation to smooth the surface normal fields and a low-dimensional embedding to recover the surface height.

The 2-D Leap-Frog algorithm is an alternative method used for solving non-linear optimizations. This algorithm can be used for reconstructing surface given three or two images. Future research can investigate several schemes to improve the result of photometric stereo using the algorithm, such as examining the snapshot size or optimization methods used in each local optimization.

REFERENCES

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