A FUZZY LOW-PASS FILTER FOR IMAGE NOISE REDUCTION

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ABSTRACT
Digital imaging devices become popular and have replaced analog ones since the invention and development of digital signal processing technology several years ago. A common problem in digital image is noise, which is often happened when the imaging devices such as digital camera, cctv, camcorder, are used in a poor lighting scene. To overcome this problem, many methods have been introduced. We proposed a new fuzzy low-pass filter to reduce additive noise in digital color image. This method is based on the low-pass filter concept in electronics engineering, which is combined with fuzzy theory. The experiment of several benchmark images shows the increased image quality and PSNR value, compared to the state-of-the-art filters. This method could be a potential alternative to be implemented in digital imaging devices or image processing activities using computers.

Keywords: fuzzy, low-pass filter, noise, image processing.

1 INTRODUCTION
As the growth of digital imaging devices where the resolutions become higher, better image quality and the speed of capturing image raised, the noise disturbance is still apparently occurred, particularly when the location of the taken image object is in dark or insufficient light. There are three common types of noise introduced in image processing, i.e. impulse noise, additive noise, and multiplicative noise. Impulse noise (e.g. salt and pepper noise) is some impulse signals which are randomly distributed on a digital image, while additive noise (e.g. Gaussian noise) is a random magnitude of signals where distribute in Gaussian distribution on an image. Multiplicative noise could be a multiplication or convolution of several noises, which are different in intensity, distribution or noise signal magnitude.

Some fuzzy methods to overcome noise problem in digital image have been introduced. Farbiz, et al. proposed fuzzy logic control to filter image [1]. Dimitri Van De Ville proposed fuzzy-based filter for Gaussian noise [2]. Pu-Yin Liu in 2004 introduced some fuzzy techniques for Gaussian and impulse noise removal [3]. Fuzzy filter for Gaussian noise in grayscale image was proposed by Mike Nachtegaele [4]. Schulte et al. proposed fuzzy-shrink wavelet method [5], and Fuzzy Color preserving Gaussian noise reduction method (FCG) in 2007 [6], [7].

In spite of many methods has been introduced, research on image denoising technique is still interesting to increase quality, image manipulations and arts, in line with the development of imaging software and technology in the world. For this reason, by following Schulte in the last mentioned method, we proposed a new fuzzy low-pass filter to reduce additive noise in digital color image.

Experiments of our work to several benchmark images smudged by additive noise, show quality increased both visually and quantitatively, which is determined by PSNR (Peak Signal to Noise Ratio) value. This new method could be potentially adopted by some imaging technology and devices for image enhancement and contribute to image processing research in the world.

This paper is organized as follow: the next chapter will explain the design of the proposed fuzzy low-pass filter. Chapter 3 describes the experiments and results of some images, and the last chapter is conclusion and discussion as the closing remarks.

2 FUZZY LOW-PASS FILTER

For the need of mathematical formulation, digital color images are represented by three \(m \times n\) \(\{m, n \in N\}\) pixel color matrix, i.e., red, green, and blue, where \(m\) and \(n\) corresponds to width and height of the image in pixels. If a clean image on
each pixel position \((i, j)\) where \(\{i, j \in N\}\) is represented by \(C(i, j, c)\), and noise component is \(\rho_c\), where \(c \in \{1, 2, 3\}\) is representing color index, i.e. 1 for red, 2 for green, and 3 for blue, then a corrupted image with additive noise \(N(i, j, c)\) can be modeled as

\[
\begin{bmatrix}
N(i, j, 1) & N(i, j, 2) & N(i, j, 3)
\end{bmatrix} = \begin{bmatrix} C(i, j, 1) + \rho_1 & (C(i, j, 2) + \rho_2) & (C(i, j, 3) + \rho_3) \end{bmatrix}
\]

(1)

In image processing, the noise component is usually modeled with some mathematical equations. For additive noise, common model used is Gaussian distribution noise, which uses normal distribution,

\[
P(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left( -\frac{(x - \mu)^2}{2\sigma^2} \right),
\]

where \(\mu\) represents mean \((\mu \in R)\), and \(\sigma\) represents standard deviation \((\sigma \in R)\).

The first step of our proposed method is to separate each color component into single color matrix \(N(i, j, c)\) for further manipulations. Then a spatial window with dimension \(K \times K\) will be evaluated along path from \(i, j=(1, 1)\) to \((m, n)\) for each color. Distance between central pixel \((i, j)\) and neighbor pixel of each color is calculated separately as Hamming Distance, that is formulated as

\[
D_c(i, j, k, l) = |N(i+k, j+l, c) - N(i, j, c)|,
\]

(2)

where \(k, l \in [-K, ..., K : K \in N]\). The distance will be used to determine the weight for central pixel correction, by considering the correlation of all color components, by using fuzzy inference rule on Takagi-Sugeno model [8], as follow.

IF the distance \(D_r\) between the central pixel of color component red \(N(i, j, 1)\) and its neighbor pixel \(N(i+k, j+l, 1)\) is SMALL

AND the distance \(D_g\) between the central pixel of color component green \(N(i, j, 2)\) and its neighbor pixel \(N(i+k, j+l, 2)\) is SMALL

AND the distance \(D_b\) between the central pixel of color component blue \(N(i, j, 3)\) and its neighbor pixel \(N(i+k, j+l, 3)\) is SMALL

THEN the weight \(w(i+k, j+l)\) for all colors is LARGE.

To get brief of how our proposed method works, let’s assume that the distances of each pixel positions \((i+k, j+l)\) in an evaluated \(K \times K\) widow are already calculated. It is not easy to surely determine how small the distance is, what parameters are used and how objective the decision to prescribe whether a distance is small or not. For this uncertainty, fuzzy concept is an appropriate choice.

In fuzzy theory introduced by Zadeh [9], class of object is determined as a continuum of grades of membership. Then the grade of object SMALL in this method will be calculated using a membership function. Considering that the small distance will be taken into account to determine the weights, the term of low-pass filter is appropriate, i.e. the smallest distance of a certain pixel to its center, the highest weights will be applied. Otherwise, larger distance will imply a lower weight for correction factor.

To determine membership function that fits to this concept, we adopting the low-pass filter (LPF) theory in electronics engineering, e.g. noise control in audio. Conceptually, all low frequency signals are passed by filter, and high frequency signals are rejected [10].

Since an ideal LPF is never found in real electronic component [11], a cut-off frequency is theoretically determined where the power of a signal is half of the maximum, or the gain voltage is about \(1/\sqrt{2}\) of the pass band. The cut off frequency is also known as the -3 dB point, because the voltage \(1/\sqrt{2}\) is equal to -3 decibel. Figure 1 shows the illustration of LPF transfer function in electronics, ideal LPF (a) and the real possible LPF approach ideal (b).

![Figure 1. Illustration of Low-Pass Filter transfer function](image-url)
Two state-of-the-art membership functions which approach ideal LPF transfer function, i.e. Z-MF and Sigmoid-MF [12], are investigated to be applied to our fuzzy filter. Varying membership function parameters will change slope and cut off. In this case, power (y-axes) is the degree of the distance (close or far), the slope of the curve determine how much distance grade will be passed by filter, which later correspond to its weighting function. Figure 2 illustrates the transfer function of selected member-ship function.

\[
\mu_s(x) = \begin{cases} 
1, & \text{if } x < a \\
1 - \frac{(x-a)^2}{(b-a)^2}, & \text{if } a \leq x \leq \frac{a+b}{2} \\
\frac{(b-x)^2}{(b-a)^2}, & \text{if } \frac{a+b}{2} \leq x \leq b \\
0, & \text{if } x > b 
\end{cases}
\]

(4)

Referring to equation (4), for Z-MF membership function, the slope will be determined by value \(a\) and \(b\), where \(a = p - \lambda\) and \(b = p + \lambda\) \(\{\lambda \in R\}\). It is also possible to change the slope for seeking the best performance by change the value of \(\lambda\). Meanwhile for Sigmoid-MF in equation (5), the slope is determined by value of variable \(a\). Later, we find that sigmoid-MF is the optimal membership function compared to Z-MF, with better result and computation time.

Weighting Calculation

Next step in this method is to calculate the weight for pixel correction in the evaluated spatial window. To perform this task, the distance degree of each color pixel in window is calculated by using equation (3) or (4), depends on what type of membership function is selected.

Then weighting function is determined by applying fuzzy IF THEN rule as mentioned before. Considering the rule, there is an intersection among the three fuzzy sets of color red, green and blue. T-norm will be the appropriate operations to aggregate the fuzzy sets intersection. There are some common operators T-norm [12], [13], e.g. minimum value (intersection operator), algebraic product, bounded product and drastic product. For this proposed method, algebraic product T-norm as formulated in (6) is selected, which implied a quantitative degree of fuzzy sets intersection between 0 and 1 to be used as weighting calculation.

\[
w_{rgb}(i+k,j+l) = \mu_r(D_r) \cdot \mu_g(D_g) \cdot \mu_b(D_b)
\]

(6)

From equation (6), the weight for each pixel position in window is derived from the SMALL distance degree \(\mu_s(D_s)\) of the three color channels respectively. This weight is applied to all color channels for the same pixel position \((i, j, k, l)\). Then a defuzzification process will result a corrected pixel for the position \((i, j)\).
Fuzzy LPF Output

Defuzzification process is the last step in this fuzzy filter. Central pixel correction of a spatial window is resulted by applying weighted average equation (7) from TSK-method (Takagi-Sugeno-Kang) to the three colors component separately.

\[
F(i, j, c) = \frac{\sum_{k=-K}^{K} \sum_{l=-K}^{K} w_{op}(i+k, j+l) \cdot N(i+k, j+l, c)}{\sum_{k=-K}^{K} \sum_{l=-K}^{K} w_{op}(i+k, j+l)}
\]

……..(7)

This calculation will result a crisp value of corrected at position \((i, j)\). After all pixels in the evaluated image are corrected, a noise reduced image is derived.

3 RESULT

Experiments of two 512x512 pixels benchmark images in Figure 3, i.e. Baboon and Lena were performed on several levels of Gaussian noise. The proposed filters i.e. LPF with Sigmoid-MF (LPF-S) and LPF with Z-MF (LPF-Z) are used to reduce these noise levels. The results then compared to the FCG subfilter 1 (FCG 1) and subfilter 2 (FCG 2) separately, which proposed by Schulte [6], and some state-of-the-art methods, e.g. median filter (MED), mean filter (or averaging filter) and center-weighted averaging filter (CWA).

The CWA (center-weighted averaging) filter is developed from mean filter, where each central pixel of evaluated spatial windows given a crisp weight (in this experiment, the weight is 4), while other neighbors only have the weight of 1. Then the sum of all pixels value in that window is divided into the total weight. Basically, this idea is the same as what fuzzy LPF do, giving a certain weight in a degree of membership function of small to each pixel in window.

\[
\text{PSNR} = 10 \log_{10} \left( \frac{255^2}{MSE(C, F)} \right),
\]

\[
MSE(C, F) = \frac{1}{3NM} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{c=1}^{C} (C(i, j, c) - F(i, j, c))^2
\]

where \(C\) and \(F\) are clean and filtered image respectively with size \(N\times M\), and \(c\) is image color index.

As seen in Figure 4, the proposed filters give more clean and sharp results, which are shown by the edge and color artifact that successfully preserved. All state-of-the-art filters (median filter mean and center-weighted averaging filter) are also seen to be successful in reducing noise in certain quality, relatively compared to the original image. The FCG subfilters proposed in [6] and [7] which works independently, are seen to be unsuccessful in reducing noise, since the FCG subfilter 1 only reduce a small portion of noise, and FCG subfilter 2 is seen to lose the image artifacts, i.e. color, edge and sharpness.
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For Lena image, when a high level noise is applied to the original image, visually the filters could overcome the noise in a certain quality. But it could still be observed that all the filters do not satisfy our preferences. A smoothing filter seems to be needed to increase output image quality.

Quantitative analysis of the tested filters shows that the proposed methods give significant results, which are placed on the top three of the filters. For Baboon image, which is rich of color and other artifacts, the proposed filter i.e. LPF-S and LPF-Z are the top three performances of the overall noise levels, which is showed in Table 1 above. Moreover, LPF-S (with sigmoid MF) could reach the overall performance. It is believed that these filters could works well on other colorful images.

Table 1. PSNR value for Baboon Image

<table>
<thead>
<tr>
<th>Image Source</th>
<th>PSNR Value (dB)</th>
<th>σ = 10</th>
<th>σ = 20</th>
<th>σ = 30</th>
<th>σ = 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noisy</td>
<td></td>
<td>28.18</td>
<td>22.22</td>
<td>18.82</td>
<td>16.46</td>
</tr>
<tr>
<td>FCG Sub1</td>
<td></td>
<td>29.39</td>
<td>23.44</td>
<td>19.95</td>
<td>17.53</td>
</tr>
<tr>
<td>FCG Sub2</td>
<td></td>
<td>18.13</td>
<td>17.66</td>
<td>16.99</td>
<td>16.26</td>
</tr>
<tr>
<td>LPF-S</td>
<td></td>
<td>26.45</td>
<td>25.32</td>
<td>24.00</td>
<td>22.64</td>
</tr>
<tr>
<td>LPF-Z</td>
<td></td>
<td>25.87</td>
<td>24.83</td>
<td>23.58</td>
<td>22.33</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>24.11</td>
<td>23.57</td>
<td>22.81</td>
<td>21.95</td>
</tr>
<tr>
<td>Median</td>
<td></td>
<td>24.11</td>
<td>23.57</td>
<td>22.81</td>
<td>21.95</td>
</tr>
<tr>
<td>CWA</td>
<td></td>
<td>26.30</td>
<td>25.07</td>
<td>23.62</td>
<td>22.19</td>
</tr>
</tbody>
</table>

Table 2. PSNR value for Lena Image

<table>
<thead>
<tr>
<th>Image Source</th>
<th>PSNR Value (dB)</th>
<th>σ = 10</th>
<th>σ = 20</th>
<th>σ = 30</th>
<th>σ = 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noisy</td>
<td></td>
<td>28.17</td>
<td>22.24</td>
<td>18.87</td>
<td>16.54</td>
</tr>
<tr>
<td>FCG Sub1</td>
<td></td>
<td>29.28</td>
<td>23.32</td>
<td>19.93</td>
<td>17.57</td>
</tr>
<tr>
<td>FCG Sub2</td>
<td></td>
<td>26.31</td>
<td>23.83</td>
<td>21.56</td>
<td>21.95</td>
</tr>
<tr>
<td>LPF-S</td>
<td></td>
<td>32.48</td>
<td>29.53</td>
<td>26.86</td>
<td>24.66</td>
</tr>
<tr>
<td>LPF-Z</td>
<td></td>
<td>32.49</td>
<td>29.28</td>
<td>26.67</td>
<td>24.58</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>31.46</td>
<td>29.12</td>
<td>26.89</td>
<td>24.99</td>
</tr>
<tr>
<td>Median</td>
<td></td>
<td>31.46</td>
<td>29.12</td>
<td>26.89</td>
<td>24.99</td>
</tr>
<tr>
<td>CWA</td>
<td></td>
<td>32.50</td>
<td>28.80</td>
<td>25.97</td>
<td>23.81</td>
</tr>
</tbody>
</table>

Meanwhile, for the Lena image with less color artifact, the proposed method is still in the top three of the overall performance. The proposed method has showed the stability performance if it could not be the best, since other methods sometimes good in low level noise, but worst in high level noise or vice versa. This stability is needed since in the real applications, we cannot predict how is the noise level could be occurred.

Overall, from Figure 6 and Figure 7, we provide an objective measure of filtered image compare to noisy image. It is clear that the proposed
method could have competitive results that potentially used in image industry for its stability and good results.

4 CONCLUSION AND DISCUSSION

A new fuzzy low-pass filter is proposed to reduce additive noise in digital color image. Low-pass filter concept in electronics engineering is adopted and applied to determine the weight for pixel correction. Experiment results show a significant stability in performance, and consistent high PSNR value when several levels of noise is applied. This idea is potential to be used as a future method in noise reduction task of imaging software or devices.

Although for the two benchmark images this method already performs a significant result both on output image quality and quantitative measurement, we believe that this filter will be well perform for any kind of images and other type of noises.

REFERENCES


